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An Investigation on Functional Relationship for Sugarcane Response to Nitrogen*

by

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Among the various means by which agricultural production can be stepped up, proper use of fertiliser is probably the pivot. In recent years, the use of chemical fertilisers has gained importance necessitating research on judicious and optimum use of fertiliser. The fertiliser - crop yield relationship expressed in the form of a mathematical function with the data obtained from trials conducted in experimental farms or cultivators' fields provides not only information on crop response to fertiliser but also enables us to specify optimum doses. Choice of suitable functional models for describing the fertiliser - crop yield relationship is therefore one of the important steps in fertiliser research. The object of this paper is to examine the suitability of different mathematical models for describing response of sugarcane to nitrogen.

Types of response functions considered: Functional relationship between crop yield and nutrient factor of production generally conform with the principle of diminishing returns and hence only functions exhibiting diministing returns are considered here.

The Spillman function, $Y=m-ar^X$, is of an exponential type wherein 'm' refers the maximum total yield attainable by increasing the nutrient input, the constant 'a' denotes the maximum response attainable from use of the input factor 'x' and the coefficient 'r' defines the ratio by which marginal productivity of the nutrient declines. The yield according to the function is assumed to approach a maximum but never attain a specific maximum nor allow negative increments of yield. The marginal products derived from this function bear a fixed ratio to each other.

The quadratic function, $Y=a+bx+cx^2$, ordinarily allows negative returns but does not impose strict restraints on the function as that of Spillman function and the marginal products do not bear a fixed ratio to each other. The coefficient 'b' and 'c' denote the linear and quadratic effect respectively of the factor input.

The quadratic square-root function, $Y=a+b\sqrt{x}+cx$, is similar to the quadratic function with $\sqrt{x}$ appearing in place of X in the quadratic expression.

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The function allows diminishing yield for successive increments of input. Unlike the quadratic function, the marginal products decline at a diminishing rate in this model.

The Cobb-Douglas function, $Y = ax^b$, where 'a' is a constant and 'b' is the elasticity coefficient measuring the importance of fertiliser to the crop allows either increasing, constant or decreasing marginal productivity but cannot account for declining yield with increased doses of nitrogen. The function assumes constant elasticity of response over the entire range of data.

The Gompertz curve, $Y = a^{(m - ar^x)}$, is a modified Spillman equation. It allows for increasing yields at an increasing rate at the lower levels of input factor which is a special quality in this function.

The function, $Y = a - \beta p^x$, is an asymptotic regression curve similar to that of Spillman. This differs from the Spillman function in that the curve of this equation is estimated by application of orthogonal polynominals. The total yield curve of this equation is asymptotic to 'a' and the marginal product obtainable from the equation never approaches zero.

Method of fitting: The fitting of Spillman, Cobb-Douglas and Gompertz functions involved transformations of the functional models into linear forms and estimation of the parameters by the least squares method. Such transformation of the functional model is generally practised in place of iterative estimation which consumes a substantial amount of computer time. The experiment considered in the present analysis included only equispaced nutrient inputs facilitating estimation of quadratic function by the method of orthogonal polynominals. The method of least squares was adopted for estimating the parameters of the quadratic square-root function. The fitting of the asymptotic regression curve was carried out by first estimating the parameter 'p' by use of orthogonal polynomial and later applying the least squares estimates to determine the constant 'β'.

Data studied. The data for the present investigation was obtained from an experiment conducted during 1960-61 in South Arcot District of the Madras State on Co. 419 sugarcane, with nitrogen applied at 11 equispaced rates. The rates of nitrogen tried were 0, 50, 100, 150, 200, 250, 300, 350, 400, 450 and 500 lb / acre. Past experimental evidence has shown that application of nitrogen in excess to sugarcane crop leads to succulence of plant tissues and greater dilution of juice resulting in poorer recovery of sucrose. This points out every possibility of variation in the response between cane yield and sugar yield to application of nitrogen. Hence, functional relationship was attempted for the yield of sugarcane as well as sugar.

Functional estimate. The estimated curves of the six functions fitted to the data on cane yield and sugar yield are presented in Figures 1 and 2. The significance of the regression mean square tested against the deviation mean square formed a best measure for determining the adequacy of the fitted functions. All the functions tried are found to be adequately fitting to the data considered as could be seen from Table 1. As the estimated constants of the individual functions are not comparable, the sums of squares of the deviations and the co-efficients of multiple determination of the fitted funciions are taken as the basis for selection of the best fits among the functions studied.

TABLE 1. *Adequacy of the fitted functions*

Fitted function		Treatment sum of squares	Sum of squares of deviations about the regression line	Coefficient of multiple determination
A. Cane yield*		1805.84		
1. Spillman	$Y=56\ 27-(38.47)\ (0.9917)^x$		113.43	0.9372
2. Quadratic	$Y=19.23+0.1880\ x-0\ 000236x^2$		188.64	0.8955
3. Quadratic square-root	$Y=15.65+2.65\sqrt{x}-0.0316x$		137.17	0.9240
4. Cobb-Douglas	$Y=16.25x^{0.2005}$		192.92	0.8932
5. Gompertz	$Y=e^{1.75-(0.4953)\ (0.9888)^x}$		102 75	0.9431
6. Asymptotic	$Y=58.81-(42.15)\ (0.7067)^x$		93.32	0.9483
B. Sugar yield*		15.67		
1. Spillman	$Y=5.46-3.34\ (0.9860)^x$		3.43	0.7810
2. Quadratic	$Y=2\ 60+0.0166x-0.000022\ x^2$		4.49	0.7132
3. Quadratic square-root	$Y=1.97+0.31\sqrt{x}-0.0065x$		3.60	0.7703
4. Cobb-Douglas	$Y=2.05x^{0.1654}$		3.80	0.7573
5. Gompertz	$Y=e^{0.7371-(0.4108)\ (0.9842)^x}$		3.20	0.7955
6. Asymptotic	$Y=5.71-(3.50)\ (0.6455)^x$		3.35	0.7861

*All the functions are significant at 1% probability level.

The asymptotic regression curve, $Y=a-\beta\rho^x$, has the lowest sum of squares of deviations and the highest co-efficient of multiple determination followed by Gompertz and Spillman the case of functions fitted to the data on sugar yield, the Gompertz seemed to have the lowest sum of squares of deviations and highest co-efficient of multiple determination. The quadratic square-root, quadratic and Cobb-Douglas functions have the largest sum of squares of deviations in both cane yield and sugar yield. All the functions fitted the cane yield data explained more than 93% of the total variation except the regression estimates of the quadratic square-root, quadratic and Cobb-Douglas functions. The functions estimated with the sugar yield data, however, accounted for only

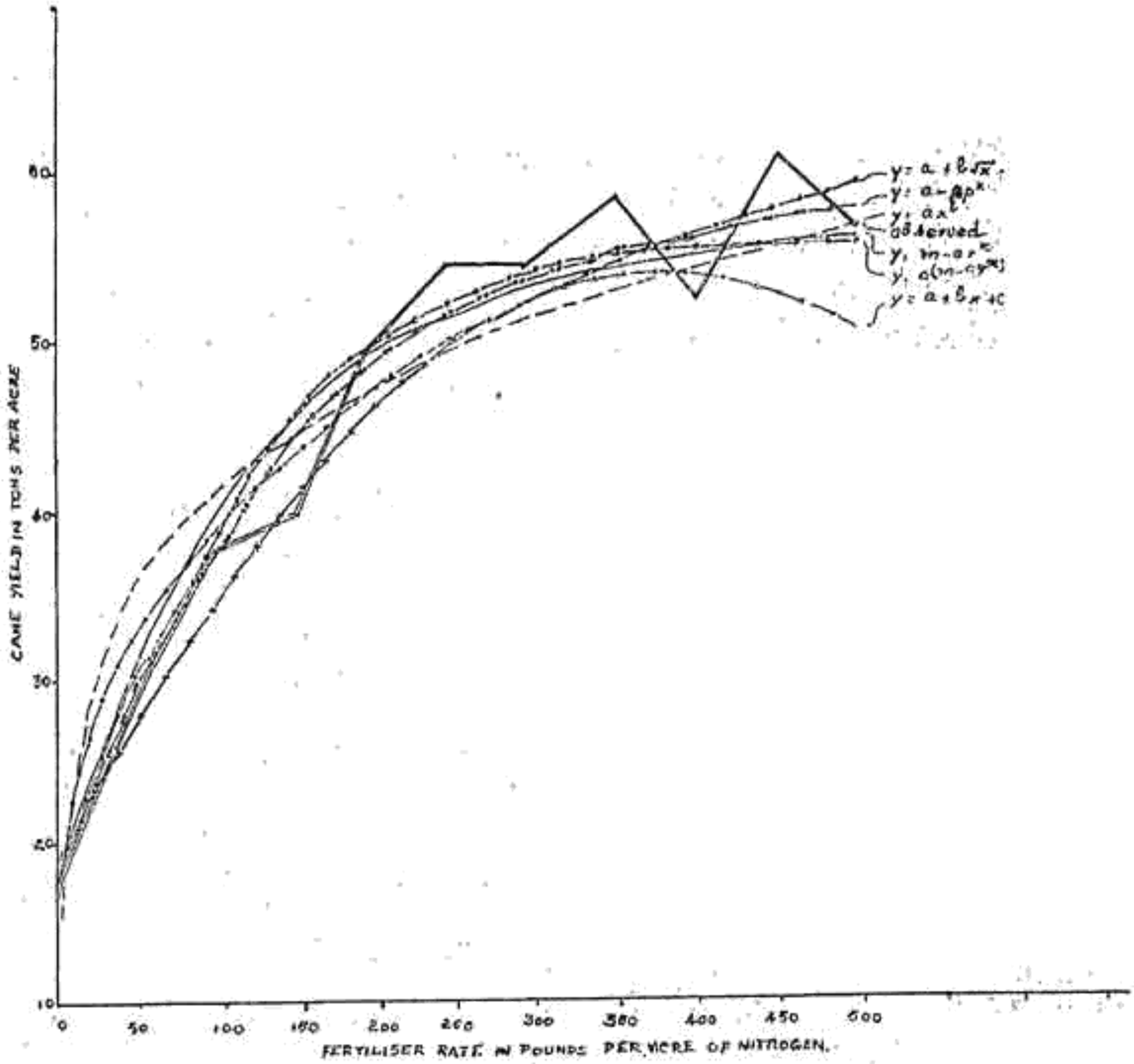

FIGURE. I. RESPONSE CURVES (CANE YIELD).

71 to 80% of the total variation, the quadratic square-root, quadratic and Cobb-Douglas functions explaining only 77%, 76% and 71% respectively of the total variation. Thus an examination of the table reveals high suitability of the three functions other than quadratic square-root, quadratic and Cobb-Douglas functions.

Estimates of the most profitable rates. The primary objective of the fitted response function is to determine the economic optimum which will maximize the profit of the farmer under a given factor-product price ratio. This most profitable rate could be obtained by equating the marginal cost of the input factor to the marginal value of the produce as given by the equation

$$p\frac{dy}{dx}=q$$

where 'p' is the price per unit of the produce and 'q' is the price per unit of the input factor and dy/dx is the marginal productivity of the input.

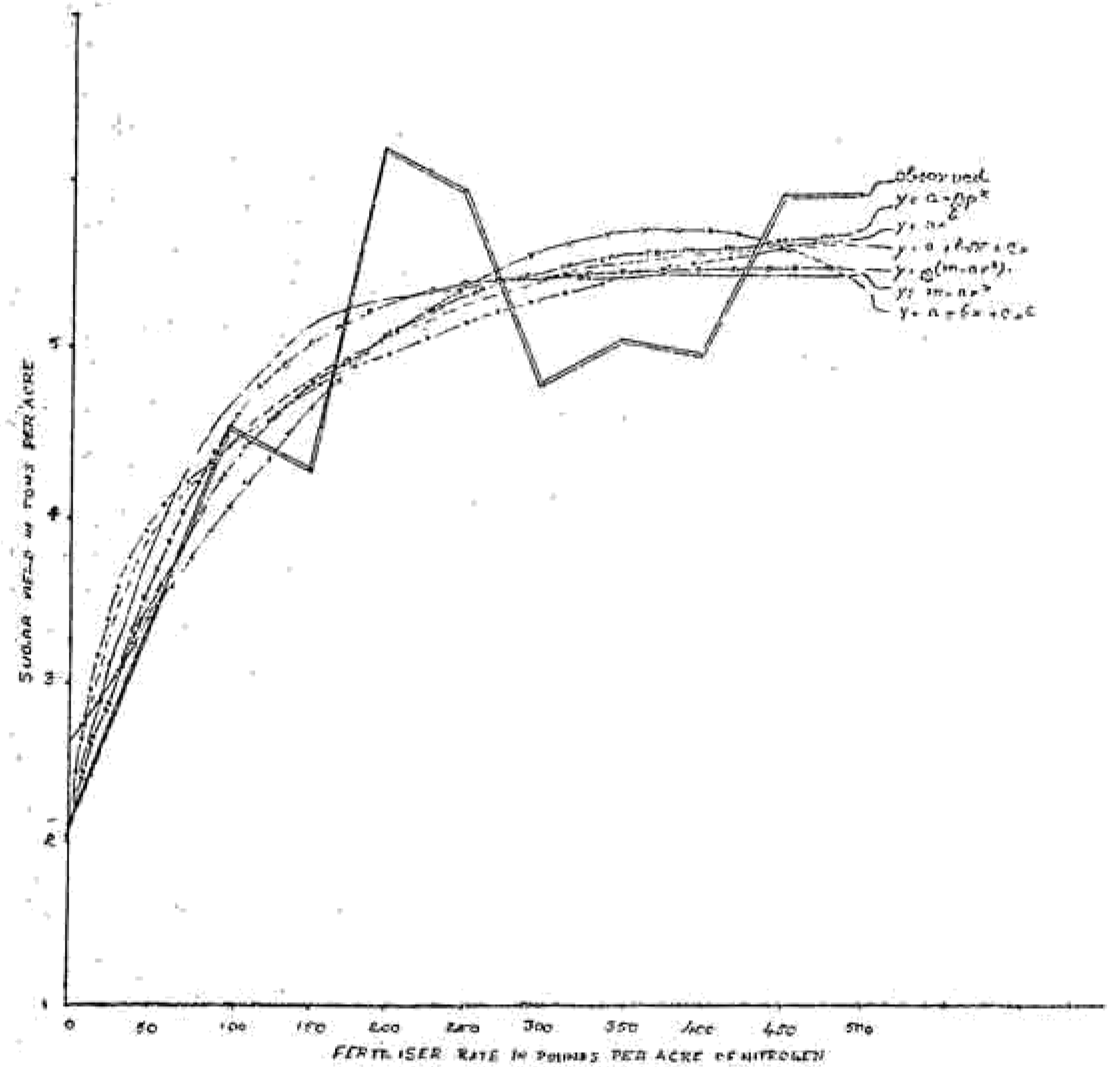

FIGURE 2. RESPONSE CURVES (SUGAR YIELD)

The estimated economic optima for all the functions are given in Table 2. The estimates of the most profitable rate are found to vary considerably from function to function. The estimates of the quadratic square-root, Cobb-Douglas and the asymptotic regression functions fitted to cane yield data are far above the doses of nutrient tried and hence they are not of much value. Similarly the asymptotic curve and the Gompertz function fitted to the sugar yield data estimated highly extrapolated rates. The most profitable rates obtained from Spillman and quadratic functions only are well within the range of nutrient levels tried in both the types of data considered in our analysis. The predicted yields at the most profitable rates of the nutrient, however, do not exhibit considerable variation except in the case of Cobb-Douglas function which has recorded 62.82 tons/acre as the maximum cane yield and 7.12 tons/acre of sugar yield. The net returns due to the application of nitrogen at the most profitable rates have also been computed and presented in Table 2. This is the value of the increased yield above the yield estimated at zero level of nutrient applied. The Cobb-Douglas function has given the highest net returns in both the cases. No outstanding differences are observed between the estimated net returns of the Spillman quadratic and asymptotic regression functions.

TABLE 2. *Economic estimates of the fitted functions.*

	Predicted yield at the zero application of nitrogen (Tons/acre)	Total returns at the most profitable rate		Increase in returns due to application of nitrogen		Nitrogen applied at the most profitable rate (lb/acre)	Cost of nitrogen (Rs./acre)	Net returns due to nitrogen (Rs./acre)
		(Tons/ acre)	(Rs / acre)	(Tons/ acre)	(Rs./ acre			
A. Cane yield*								
1. $Y=m-ar^x$	17.80	52.10	2969.70	34.30	1955.10	268.00	227.80	1727.30
2. $Y=a+bx+cx^2$	19.23	56.44	3217.08	37.21	2120.97	366.74	311.73	1809.24
3. $Y=a+b\sqrt{x}+cx$	15.65	49.84	2840.88	34.19	1948.83	811.95	690.16	1258.67
4. $Y=ax^b$	16.25	62.82	3580.74	46.57	2654.49	850.00	722.50	1931.99
5. $Y=a-\beta p^x$	16.66	58.71	3346.47	42.05	2396.85	873.00	742.05	1654.80
B. Sugar yield*								
1. $Y=m-ar^x$	2.12	5.35	7222.50	3.23	4360.50	246.00	209.10	4151.40
2. $Y=a+bx+cx^2$	2.60	5.73	7735.50	3.13	4225.50	362.95	308.51	3916.99
3. $Y=a+b\sqrt{x}+cx$	1.97	5.64	7614.00	3.67	4954.50	472.63	401.74	4552.76
4. $Y=ax^b$	2.05	7.12	9512.00	5.07	6844.50	1850.00	1572.50	5272.00
5. $Y=a-\beta p^x$	2.21	5.71	7708.50	3.50	4725.00	795.00	675.75	4049.25
C. Sugar yield**								
1. $Y=m-ar^x$	2.12	5.18	2590.00	3.06	1530.00	177.00	150.45	1379.55
2. $Y=a+bx+cx^2$	2.60	5.70	2850.00	3.10	1550.00	338.64	287.84	1262.16
3. $Y=a+b\sqrt{x}+cx$	1.97	5.51	2755.00	3.54	1770.00	357.21	303.63	1466.37
4. $Y=ax^b$	2.05	5.87	2935.00	3.82	1910.00	575.00	488.75	1421.25
5. $Y=a-\beta p^x$	2.21	5.70	2850.00	3.49	1745.00	682.00	579.70	1165.30

*The assumed price of one pound nitrogen is Re. 0.85, and one ton of harvested cane is Rs. 57. The price assumed for one ton of sugar is Rs. 1350.

**The price payable by the sugar mills for the cane supplied by the farmers on sugar recovery basis is as Rs. 500 per ton of sugar.

Usefulness of the economic estimates. The farmers' interest lies in the recommendation that will maximize his profits under the prevailing price situation. The economic estimates of the Spillman and quadratic functions of the cane yield data indicate that the cultivator can achieve a maximum of 52–56 tons of cane yield if he is interested in maximising his profit. The produce of the sugarcane crop is generally disposed to sugar mills as wet cane. The economic dose of 268 lb of nitrogen and 367 lb of nitrogen obtained from Spillman and quadratic functions respectively of the cane yield data is applicable only to those farmers who supply cane to sugar mills on the basis weight of cane. For others who supply cane to factory on sugar recovery basis, economic optima derived from functions estimated with sugar yield data are applicable. Assuming a price of Rs. 500/- per ton of sugar recoverable from the cane supplied by the farmers, economic doses of 177 lb of nitrogen and 339 lb of nitrogen are obtained for the Spillman and quadratic functions respectively. The economic optima recommended by the set of funcions on the sugar yield data as well as the net returns attainable due to application of nitrogen are observed to be considerably lower than those of the cane yield data. From the farmers' point of view, therefore, supply of cane to sugar mills on the basis of weight is preferable under the assumed price conditions.

Discussion: All the six functions that were tried in the present study are found to be adequate fits. Among the fitted functions, the Spillman, Gompertz and asymptotic regression equations are observed to be best fitting as they stand as a group with the lowest sums of squares of deviations and highest coefficients of multiple determination. However, on the basis of this criteria alone, the adaptability of other functions cannot be ruled out in as much as these functions also are found to be adequately fitting to the data. The coefficients of deviations of these three best fitting functions do not show appreciable differences among them and therefore, with little variations between them, a preference of one function over the other might be based on the complexity of estimation of the function. The Spillman function can be preferred to the other two functions considering the complex computational procedure in the estimation of these two functions. Further, the estimate of economic optima is difficult to obtain in the case of Gompertz. The most profitable rate obtained for the asymptotic regression curve is also not useful as they are found to be extremely high rates-much greater than reason would permit. The final produce of the sugarcane crop is the actively growing vegetative part unlike the cereals where grains are the ultimate produce. The application of nitrogen normally activate vegetative growth of the crop and therefore, response of sugarcane crop to nitrogen can be expected to be different from that of cereals where negative returns appear at considerably lower magnitudes of the input factor. The suitability of the Spillman function indicates absence of negative returns in the present analysis. Possibly the magnitude of the input factor tested in our analysis would not have been large enough to cause decline in the total returns. A further experimentation with still higher rates of nitrogen might probably be useful in confirming the suitability of the functional models for sugarcane response to nitrogen.

The economic estimates of the various function reveal considerable variations among them and do not allow for any generalization. The Spillman and quadratic functions have estimated optimum doses well within the range of levels tried, thereby favouring these two functions in preference to others. The economic estimates of the functions fitted to cane yield and sugar yield data indicate that optimum level is reached at considerably lower levels of the nutrient input in the latter case. It has been observed in the past that application of nitrogen to sugarcane crop at high rates leads to reduction in the quality of juice rendering poor sugar recovery. The present analysis seems to confirm this view.

Summary and Conclusions: An investigation on functional models with the data of an experiment on sugarcane indicates high suitability of the Spillm·n and quadratic functions. All the other functions, though found to have adequate fits, could not provide useful estimate of the most profitable rate. The Cobb-Douglas function is observed to be least suitable if considered in terms of coefficient of multiple determination as well as economic optimum. The superiority of the Spillman function over the other functions provide evidence that the magnitude of the input factor tried is not large enough to allow decline in yield, if one should expect sugarcane crop to reflect diminish-increments for successive doses of nutrient application. A comparison of economic estimates observed under the assumed price restrictions indicates that the sugarcane farmers stand to gain if they supply cane to sugar mills on the basis of weight of cane rather than on the basis of sugar recovery.

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