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https://doi.org/10.29321/MAJ.10.A03775

A Note on the Methodological Considerations in Fitting the Cobb-Douglas Function to Data from Samples of Sugarcane Farms of Queensland-Australia

by

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Introduction: The production function approach to farm management problems is undoubtedly a significant step in the direction of greater application of economic theory and statistical methods, to the problems of individual farms and groups of farms, and therefore can be considered a refinement over the traditional methods. A survey of literature, however, reveals the inadequacy of work on production functions, both in India and elsewhere. This may be due to the fact that the origin and development of this approach is more recent, or because of some of the important limitations involved in such studies, particularly those concerning farm surveys. Within the approach again, there seems to be the belief that while biological

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Received on 3—8—1967.

or physical production functions derived from experimental data produce more useful information on problems like fertiliser application, feed intake and milk output, feed and pork production, *etc.*, work on farm functions appears to be as yet limited in scope. This is perhaps due to the difference in approach of the two methods, the most important of which is the fact that, while the experimentor has control over the major factor imputs in the physical functions enabling him to manipulate the quantities and types, no such thing is possible in the case of farm functions where the investigator deals, ex-post, with data produced by individual decisions of a large number of managers. The obvious result of this is the limited range to which observations can be confined in the farm functions, than would be economically allowable. To this has to be added the next important problems of the choice of a suitable form of function, which appears to be difficult at first, but is not quite so because of the limited number of the more generally accepted types of functions. This difficulty applies both to the biological function as well as farm functions. The merits and demerits of these functions are described, though briefly, by McCarthy (1959) and others in their various publications. Be that as it may, my immediate interest is the Cobb-Douglas type of production which has been used most frequently to express input-output relationships of firms, and which is the one used by me in my study of the sugarcane farms of Queensland, Australia.

The Function Used: An attempt is made in this paper to evaluate methodology underlying results obtained from a study of two samples of sugarcane farms from Ayr District (irrigated farms) and Mackay District (dry farms). The function used is the modified Cobb-Douglas type of the following form,

$$A = aB^{b_1}\ C^{b_2}\ D^{b_3}\ E^{b_4}$$

where A represents gross income; B, C, D and E are factor inputs; a is a constant, and b_1, b_2, b_3 and b_4 are the elasticities of the product with respect to the inputs B, C, D and E respectively.

The data is expressed logarithmically and fitted to the function by least square multiple regression method.

Clarke (1954) pointed out some of the major limitations of the production function approach. The major inherent limitations according to him are,

(1) possible inappropriateness of the function used to represent the data,

(2) impossibility of obtaining variables that are wholly separate, distinct and additive,

(3) impossibility of inputs being perfectly substitutive and divisible,

(4) inability to take ready account of management,

(5) impossibility of obtaining clear homogeneity of the sample.

More specific limitations are,

(6) the inability to keep the number of variables down and yet independent,

(7) the large differences in results due to small variations in measurement of variables.

Reder (1943) distinguishes between the production function of economic theory which he calls intra-firm functions and the iudustry functions which are called inter-firm functions. He says firstly, that the intra-firm or the physical production function, shows the functional relationship between the input quantities and the output of a firm, secondly the intra-firm function is in physical quantities while the industry function represents the value added in manufacturing to physical units of labour and capital, thirdly the marginal value productivities derived from the industry functions can be obtained directly from the function, while in the intra-firm function it is the partial derivative of total product in respect to that particular factor multiplied times the marginal revenue for this output.

Bronfenbrenner (1944) stated that the marginal productivity of the factor could be the same for all firms with different production functions, in which case the function fitted by Douglas is inappropriate. His point is that, under perfect competition and under equilibrium conditions the slope of the function should be same for industries and firms.

Smith (1945) observed that under conditions of perfect competition and perfect factor markets, a long run statistical function from cross sectional data should allow estimates of the conventional intra-firm marginal value productivities. Under these conditions the average and marginal value productivities should equal the factor prices paid by each firm. These observations appear to be more relevant to function derived from farm survey data.

Plaxico (1955) suggests that "Marginal value productivity estimates derived from the Cobb-Douglas type function can be seriously biased by non-optimum aggregation of inputs or by non-optimum aggregation of outputs".

In the light of the problems raised above, the various advantages of using the Cobb-Douglas function are stated below, before an attempt is made to explain how the sugar data gets over many of the limitations and objections.

Advantages of the Cobb-Douglas function:

(a) Its use involves relatively simple computations fitting in well with the time and resources available.

(b) The function gives immediately, through the regression coefficients, elasticities of production of the individual factors, and these elasticities are independent of the unit of measurement. (This is important in the study taken up by the author since all the inputs are not mesured in value terms).

(c) The function permits automatically the phenomenon of diminishing marginal productivity of each factor input and causes the productivity of one resource to be dependent on the magnitude of others, without too many degrees of freedom. To get these conditions satisfied with a quadratic function, it should include linear, squared and cross-product terms for each factor, which means with four factors we have to estimate only four regression coefficients for a Cobb-Douglas and fourteen for a quadratic function.

(d) If it is assumed that the errors in the data are small and normally distributed, a logarithmic transformation of the variables will preserve the normality to a great degree in the distribution of errors in the data.

(e) The function as used does not have a restrictive assumption regarding returns to scale, which may be decreasing, constant or increasing, as measured by the sum of elasticities of factor inputs. This allows the set of hypotheses about the nature of the returns to scale to be easily applied.

(f) Hypothesis about the size of the individual elasticities may be easily tested.

(g) The function can be easily manipulated so that marginal productivity estimates of the factors may be easily obtained.

(h) Finally, as various workers have indicated in the past, after fitting their data to a Cobb-Douglas function, that inspite of the various limitations of the function it works in practice.

Now as far as the sugar data of the present study is concerned it is the author's belief that many of the limitations pointed out by Clarke and others do not apply to it. Firstly, dealing with the appropriateness or otherwise of the function used, it is true with the kind of farm records available, and with the type of function used, the exact shape of the various production functions along their entire length or range cannot possibly be demonstrated. But it is reasonable to assume that farms of the type included in the present study are operating around the economic equilibrium point and our interest mainly lies in that small range only and not in the extremes. If this assumption is true then the Cobb-Douglas type of function is certainly the most aprropriate one.

Secondly, the problem of obtaining independent variables has not been as formidable in the sugarcane farming in Australia as in others. The four inputs selected, *viz.*, fertilisers, labour, plant and machinery and farm peak, do satisfy all requirements of the independent variables being separate, distinct and additive. It was found on examination of correlation tables that interdependence among these inputs is not large.

Thirdly, with regard to the question of substitutability and divisibility of variables, the selected variables do possess these attributes to a much greater degree, than similar ones included in previous studies. Fertiliser and plant and machinery being measured in value terms can be reduced to as small a unit as a A£, while labour can be man weeks and farm peak to a ton of cane. These, I believe, are sufficiently small for marginal calculas to be applied with advantage.

I have sound reason to assume management is relatively homogeneous. Firstly the belief that managerial ability tends to be uniform among the farms studied was verified by the author by indirect assessment and found largely to be true. Secondly, we have in sugarcane farming an industry in which there are a great number of "privileges and penalities" accruing to the individual farmer by way of (1) fixation of gross and net assigned areas on the farm, (2) allotment of farm peaks in tons of cane or sugar, (3) attention to pests and disease control, (4) stipulation of the varieties of cane to be grown, (5) directions regarding time and quantity of cane to be harvested, (6) fixation of price of cane in advance, (7) fixation of the wages of the cane cutters and other casual labour, *etc.*, that relative to other types of farming the farmer is beset with very few problems in decision making, except timeliness of operations in planting, cultivation, application of fertilisers, *etc.*

Finally to the most important limitation, that of obtaining a homogeneous sample. This is fundamental to the study since if the sample is not homogeneous, the derived function for the group can never truely indicate the underlying relationships that exist within individual farms. In other words the function loses its value as a tool of predictability, and for the projection of results to individual farms. My data show that individual farms in each of the two samples selected in the study are highly homogeneous, with regard to size, production techniques, plant and equipment and with similar soils and comparable climates. In fact, I cannot think of any other farming system in Australia or elsewhere where the circumstances surrounding any crop are so well organised and controlled as the sugar farms of Queensland. (Hence a homogeneous sample).

Coming to the points raised by Reder and Bronfenbrenner, it seems that they apply more to industry functions, where the production functions for individual firms may not be similar, where there is lack of homogeneity of sample and where pure competition is an exception rather than the rule. I believe farm management studies of the type discussed in the present investigation, do not warrant any distinction being made between inter-firm and intra-firm productivities of the factors of production. In this respect Smith's reference to perfect competition and perfect factor markets, which allow (a) a long run statistical function from cross-sectional data, (b) estimates of the conventional intra-firm marginal value productivities, appear to be more relevant to the situation in agriculture.

Heady and Dillon (1961) say that "obviously all of the problems, relating to multi-collinearity in time series data, aggregation across industries, perfect competition in the market, and others do not apply to functions derived from a sample of competetive firms such as those found in agriculture".

The problems raised by Plaxico regarding aggregation of the variables and the possible bias of the result, appear to be more fundamental and important. Although the present study did not offer much scope for aggregation of either the output or the inputs generally, the only case where it is done with respect to the input item plant and machinery, it is in accordance with the principles laid down by Johnson (1956).

The Functions: The identification of the variables is as follows:

A = Gross income (in A£s.)	$Y = \text{Log } A$
B = Fertilisers (in A£s.)	$X_1 = \text{Log } B$
C = Labour (in man weeks)	$X_2 = \text{Log } C$
D = Plant and Machinery (in A£s)	$X_3 = \text{Log } D$
E = Farm Peak (in tons of cane)	$X_4 = \text{Log } E$

The following two tables give the regression equations, R^2, sample number, mean quantity of resources and mean quantity of output of the two samples analysed.

TABLE 1. *Regression Equation*

Year 1	Sample No. 2	R^2 3	Sum of Elasticities 4	Regression equation 5
MACKAY SAMPLE				
1958	30	·7952	1·286663	$A = aB^{\cdot 036853} C^{\cdot 196947} D^{\cdot 223029} E^{\cdot 829734++}$
1959	30	·7288	0·920206	$A = aB^{\cdot 084748} C^{\cdot 012394} D^{\cdot 104912} E^{\cdot 718153++}$
1960	30	·7979	1·029365	$A = aB^{\cdot 222250+} C^{\cdot 029370} D^{\cdot 298530+} E^{\cdot 479215+}$
1961	30	·9479	1·020948	$A = aB^{\cdot 098702+} C^{-\cdot 046004} D^{\cdot 005090} E^{\cdot 963100+++}$
1958–'61	120	·8143	1·053588	$A = aB^{\cdot 113588+} C^{\cdot 036216} D^{\cdot 135163+} E^{\cdot 768619+++}$
AYR SAMPLE				
1958	25	·8943	1·236113	$A = aB^{\cdot 198378+} C^{\cdot 120789} D^{\cdot 119494} E^{\cdot 797452+++}$
1959	30	·9709	1·018918	$A = aB^{\cdot 050680+} C^{-\cdot 050751} D^{\cdot 038433} E^{\cdot 980556+++}$
1960	32	·8428	0·958008	$A = aB^{\cdot 024631} C^{\cdot 178113+} D^{-\cdot 004507} E^{\cdot 759774+++}$
1958–'60	87	·8944	1·053877	$A = aB^{\cdot 065988+} C^{\cdot 086508} D^{\cdot 032727} E^{\cdot 868651+++}$

\+ Significant at 5 per cent level

++ Significant at 1 per cent level

+++ Significant at 0·1 per cent level

TABLE 2. *Mean Quality of Resources and Mean Value of Output*

Year	Sample No.	Mean value of fertilisers (B)	Mean number of man weeks (C)	Mean value of plant and machinery (D)	Mean tons of farm peak (E)	Mean value of output (A)
		A £		A £		A £
MACKAY SAMPLE						
1958	30	659	76	3530	936	5943
1959	30	376	76	3208	936	4881
1960	30	477	76	3095	936	4832
1961	30	541	76	2577	936	5183
1958–'61	120	503	76	3083	936	5192
AYR SAMPLE						
1958	25	663	87	4432	1834	9412
1959	30	410	91	4253	1968	10380
1960	32	528	92	3793	1995	10230
1958–'60	87	517	90	4127	1938	10040

TABLE 2. *(Contd.)*

*Mean Marginal Productivities of Inputs**

Year	Sample No.	Mean value of fertilisers (B)	Mean number of man weeks (C)	Mean value of plans and machinery (D)	Mean tons of farm peak (E)	Mean value of output (A)
		A £		A £		A £
MACKAY SAMPLE						
1958		0·033	15·401	0·376	5·268	
1959		1·100	0·796	0·160	3·745	
1960		2·251	1·867	0·466	2·474	
1961		0·946	—3·137	0·010	5·333	
1958–'61		1·173	2·474	0·228	4·264	
AYR SAMPLE						
1958		2·816	13·067	0·254	4·093	
1959		1·283	—5·789	0·094	5·172	
1960		0·477	19·805	—0·012	3·896	
1958–'60		1·282	9·650	0·080	4·500	

Note: The means mentioned above for the inputs and outputs are the Geometric means.

* B — £s per one £ spent.

C — £s per man week.

D — £s per one £ spent.

E — £s per ton of cane.

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