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Quicker Methods in the Analysis of Variance

by

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Introduction: The two simplest designs for field experiments: Completely Randomised and Randomised Block, are in use in the various Agricultural Research Stations of the State. The statistical analysis of experimental data arising from these designs are done by the well known "Analysis of Variance" method developed by R. A. Fisher. Use of this technique is straight—forward and the whole analysis is easily done with the help of a calculating machine. The calculation of 'Sum of squares' in this technique of analysis is much quickened by the use of a calculator. Unfortunately, in the absence of a calculating machine, the tedious calculations are done using Tables of Squares. The object of this paper is to place before the Research workers an alternative method which involves no calculation of "Sums of Squares" and consequently is quicker and less laborious.

Materials: Field data from four experiments, two in each of completely Randomised design and of Randomised Block design conducted by the State Research Officers, have been referred to the author for statistical analysis. These data have been used to illustrate the procedures involved in the usual and the alternative methods of statistical analysis.

The alternative method requires the use of certain tables which have been published in the "Biometrika Tables for Statisticians 1954". These relevant portions of the Table are given in Appendix I Tables 1–A, 1–B and 2.

Methods and Results: (A) Completely Randomised design.

Illustration I: Indirect manuring of paddy — experiment conducted by the Government Agricultural Chemist in 1957.

(Grain yield in lb. per plot)

Replication / Treatment	1	2	3	4	5	Total	Mean
1. Control	159	131	149	161	144	744	149
2. Sesbania alone	213	199	219	223	209	1063	213
3. Sesbania + P_2O_5	214	222	233	230	212	1111	222
4. Sunhemp alone	203	209	225	200	190	1027	205
5. Sunhemp + P_2O_5	201	191	223	196	181	992	198
Total	990	952	1049	1010	936	4037	

(a) Analysis of variance method :

Analysis of Variance :

Due to	D. F.	S. S.	M. S.	F.
Between treatments	4	16365·04	4091·26	24.5
Within treatments	20	3334·20	166·71	
Total	24	19699·24		

The treatment differences are significant.

S. E./plot $= \sqrt{166 \cdot 71} = 12 \cdot 9$ lb.

(b) Alternative method :

Step 1 : Form the following table from the field data obtained.

Treatment	Range	
Control	30	
Sesbania alone	24	
Sesbania + P_2O_5	21	Total = 152
Sunhemp alone	35	Mean Range = 30·4
Sunhemp + P_2O_5	42	

The range for each treatment is the difference between the maximum and the minimum yields obtained for the treatment :

Step 2 : Enter Table 1–A with n = 5 and k = 5 and obtain

V = 18·4

C = 2·36

dn = 2·33

n = the size of the sample from which each range is calculated.

k = the number of samples.

Step 3 : Calculate 'q' difined as

$$q = \frac{(\text{Maximum Treatment mean} - \text{Minimum Treatment mean})\, c \times \sqrt{n}}{\text{Mean range}}$$

i. e.

$$q = \frac{73 \times 2 \cdot 36 \times 2 \cdot 24}{30 \cdot 4} = 12 \cdot 69$$

Step 4 : Enter Table II with n = 5 and v = 18·4 and obtain the limits for q 5% as 4·28 and 4·25.

Thus the calculated 'q' is greater than the Table value. The treatment differences are therefore significant — the same conclusion as obtained by the Analysis of variance method.

Step 5: The standard error per plot is given by

$$\frac{\text{Mean range}}{dn} \quad \text{i. e.} \quad \frac{30 \cdot 4}{2 \cdot 33} = 13 \cdot 04$$

A difference of 0·14 lb. per plot exist in the S. Es/plot calculated by the two methods.

Illustration 2: Duration studies in Pot culture with groundnut by the Groundnut Physiologist.

(Wet weight in gms. of 10 plants)

Replication / Treatment	1	2	3	4	5	6	Total	Mean
D 7	26·4	23·8	28·5	21·4	23·0	24·4	147·5	24·6
L 7	28·0	24·4	24·2	22·8	25·2	25·8	150·4	25·1
L. Naph	23·0	25·5	20·2	21·6	23·7	25·6	139·6	23·3
B. Indole	25·1	26·1	24·3	23·6	24·6	21·5	145·2	24·2
Water	19·7	26·9	25·7	20·0	20·0	21·0	133·3	22·2
Control	23·8	22·9	24·5	23·6	18·6	20·4	133·8	22·3
	146·0	149·6	147·4	133·0	135·1	138·7	849·8	

(a) Analysis of variance method:

Analysis of variance:

Due to	D. F.	S. S.	M. S.	F.
Between Treatments	5	43.16	8·63	1·61
Within treatments	30	160·68	5·35	
Total	35	203·84		

The treatments do not produce significantly different wet weights. S. E/plot or obervation $= \sqrt{5 \cdot 35} = 2 \cdot 31$

(b) Alternative method:

Step 1: Range Table:

Treatment	Range
D 7	7·1
L 7	5·2
L Naph	5·4
B. Indole	4·6
Water	7·2
Control	5·9
Total	35·4
Mean	5·9

Step 2 : n=6, k=6

From Table 1–A

v=22·6

c= 2·56

dn= 2·53

Step 3 : $q=\frac{2\cdot9\times2\cdot56\times2\cdot45}{5\cdot9}=3\cdot08$

Step 4 : Limits for $q_{5\%}$ from Table with n=6 and v=22·6 are 4·37 and 4·45.

The calculated value being less than the Table value, there is no reason to believe that the treatments tried differ significantly.

Step 5 : S. E/Observation$=\frac{5\cdot9}{2\cdot53}=2\cdot33$

Thus practically the same conclusion and the same S. E. have been obtained by the alternative method also.

B. *Randomised Block Design :*

Illustration 3 : Brinjal type collection summer 1955—experiment conducted by the Horticulturist, Coimbatore.

(Yield in ozs./plot)

Blocks / Variety	I	II	III	IV	Total	Mean
H 68	152	108	133	146	539	134·75
H 93	241	256	212	219	928	232·00
H 98	60	81	49	69	259	64·75
H 89	96	122	101	85	404	101·00
H 81	102	86	98	85	371	92·75
H 61	151	176	170	167	664	166·00
Total	802	829	763	771	3165	131·88

(a) Analysis of variance method :

Analysis of variance :

Due to	D. F.	S. S.	M. S.	F.
Blocks	3	458·2		
Varieties	5	72750·4	14550·1	58·27
Error	15	3746·1	249·7	
Total	23	76954·7		

S. E./plot$=\sqrt{249\cdot7}=15\cdot80$ ozs.

The varieties are significantly different in their yield capacities.

(b) Alternative method:

Step 1: Form the following tabe from the data presented.

Deviations from the varietal mean:

Blocks / Varieties	I	II	III	IV
68	17·25	—26·75	— 1·75	11·25
93	9·00	24·00	—20·00	–13·00
98	— 4·75	16·25	—15·75	4·25
89	— 5·00	21·00	0·00	–16·00
81	9·25	— 6·75	5·25	– 7·75
61	—15·00	10·00	4·00	1·00
Range	32·25	50·25	25·25	27·25
Total	135·00			
Mean	33·75			

To form the above table first obtain the differences of individual yields from their respective varietal means. Then, the range for each block is obtained by finding the difference between the maximum and the minimum deviation.

Step 2: Enter Table 1–B with n=6 and k=4 and obtain.

v=13·9

c=2·23

k=no. of blocks

n=no. of plots per block.

Step 3: Calculate 'q' defined as

$$q=\frac{(\text{Max. mean} - \text{Min. Mean})\ c \times \sqrt{k}}{\text{mean range}}$$

i. e.

$$q=\frac{167·25\times2·23\times2}{33·75}=33·15$$

Step 4: Enter Table II with n=6 and v=13·9 and obtain that q 5% point lies between 4·69 and 4·64. The calculated value is higher than this value. Hence the varieties are significantly different in their yield capacities.

Step 5: The S. E/plot is given by

$$\frac{\text{Mean range}}{C}=\frac{33·75}{2·23}=15·13$$

A difference of 0·67 ozs. exist in the S. Es/plot calculated by the two methods.

Illustration 4: Bhendi type collection — summer 1955 — experiment conducted by the Horticulturist, Coimbatore.

(Yield in ozs./plot)

Blocks / Varieties	I	II	III	IV	Total	Mean
H 10	45·75	62·00	49·50	54·00	211·25	52·81
H 31	64·00	62·25	38·50	43·00	207·75	51·94
H 12	47·00	40·50	30·25	32·75	150·50	37·62
H 13	46·25	56·25	62·75	34·75	200·00	50·00
H 15	59·50	52·75	66·00	47·75	226·00	56·50
H 14	52·50	72·50	25·50	48·00	198·50	49·62
Total	315·00	346·25	272·50	260·25	1194·00	

(a) Analysis of variance method

Analysis of variance:

Due to	D. F.	S. S.	M. S.	F.
Blocks	3	781·89		
Varieties	5	827·28	165·46	1·37
Error	15	1805·71	120·38	
Total	23	3414·88		

S. E/plot $= \sqrt{120 \cdot 38} = 10 \cdot 97$

There is no significant difference in the yields of the varieties.

(b) Alternative method:

Step 1:

Table of deviations from the varietal means:

Blocks / Variety	I	II	III	IV
H 10	— 7·06	9·19	— 3·31	1·19
H 31	12·06	10·31	—13·44	— 8·95
H 12	9·38	2·88	— 7·37	— 4·87
H 13	— 3·75	6·25	12·25	—15·25
H 15	3·00	— 3·75	9·50	— 8·75
H 14	2·88	22·88	—24·12	— 1·62
Range	19·22	26·23	36·37	16·44
Total	98·56			
Mean	24·64			

Step 2:

k=4 (no. of blocks)
n=6 (no. of plot/block)

From 1–B with k=4 and n=6, obtain

v=13·9
c=2·23

Step 3:

$$q=\frac{(\text{Max. Mean} - \text{Min. mean})\ c \times \sqrt{k}}{\text{Mean range}}=$$

$$=\frac{18.88 \times 2 \cdot 23 \times 2}{24 \cdot 64} = 3 \cdot 42$$

Step 4: q 5% point for n=6 and v=13·9 from Table II lies between 4·69 and 4·64. The calculated 'q' being less than this value the varietal differences in yield are not significant.

Step 5:

$$\text{S. E/plot} = \frac{\text{Mean range}}{C} = \frac{24 \cdot 64}{2 \cdot 23} = 11 \cdot 04$$

The S. Es. obtained by the two methods are practically be same.

Discussion and Conclusion: It is evident from the illustrations that the alternative methd is much less time-consuming because the ranges of values alone are calculated instead of the "Sums of squares", and the ranges are very easily calculated. The statistical conclusions arrived at are also identical.

The statistical analysis however, does not stop with the test of significance. The error involved in experiment due to unknown and uncontrolled causes marked as 'error' in the analysis of variance is further used in the comparison of treatment responses by calculating the standard errors. The S. E. per plot or per observation which is obtained by extracting the square root of the error mean square is a fundamental statistical measure. The closer the agreement on the values of the S. E. between the two methods, the greater will be the justification for the adoption of the new method.

In the illustrations 1 and 3 the values of S. E. do not agree as closely as those obtained in the illustrations 2 and 4. In the former experiments however, great variations exist in the treatment responses, while the treatment effects in experiments 2 and 4 do not differ very much. Thus, the agreement in the values of S. E. is close when the treatment differences are small. This is an important point counting in the favour of the new method, because it is only in such marginal cases the alternative method should give S. E. values as precise as the usual method so that its adoption may not lead to loss of information.

Thus, the alternative method is a good substitute for the usual method of statistical analysis. It saves time with no loss of efficiency.

Summary: An alternative method of statistical analysis of date arising from completely Randomised and Randomised Block designs is illustrated with worked out examples. The method does not involve calculations of 'Sums of Squares' required by the usual Analysis of Variance method and hence is less time-consuming and less laborious. The method also does not seem to involve any loss of efficiency.

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REFERENCES

1. Pearson E. S. & Hartley H. O. (1954) Biometrika Tables for Statisticians Vol. 1 Cambridge University Press, England.

APPENDIX I

Table I–A

Scale factor, 'c', and equivalent degrees of freedom, 'v', appropriate to a simple classification into 'k', groups of 'n' observations.

K \ n	2		3		4		5		6		7		8		9		10	
	V	C	V	C	V	C	V	C	V	C	V	C	V	C	V	C	V	C
1.	1·0	1·41	2·0	1·91	2·9	2·24	3·8	2·48	4·7	2·67	5·5	2·83	6·3	2·96	7·0	3·08	7·7	3·18
2.	1·9	1·28	3·8	1·81	5·7	2·15	7·5	2·40	9·2	2·60	10·8	2·77	12·3	2·91	13·8	3·02	15·1	3·13
3.	2·8	1·23	5·7	1·77	8·4	2·12	11·1	2·38	13·6	2·58	16·0	2·75	18·3	2·89	20·5	3·01	22·6	3·11
4.	3·7	1·21	7·5	1·75	11·2	2·11	14·7	2·37	18·1	2·57	21·3	2·74	24·4	2·88	27·3	3·00	30·1	3·10
5.	4·6	1·19	9·3	1·74	13·9	2·10	18·4	2·36	22·6	2·56	26·6	2·73	30·4	2·87	34·0	2·99	37·5	3·10
10.	9·0	1·16	18·4	1·72	27·6	2·08	36·5	2·34	44·9	2·55	52·9	2·72	60·6	2·86	67·8	2·98	74·8	3·09
dn.		1·13		1·69		2·06		2·33		2·53		2·70		2·85		2·97		3·08
C.D.	0·88		1·82		2·74		3·62		4·47		5·27		6·03		6·76		7·45	

Table 30–A of the 'Biometrika Tables for statisticians' Vol. 1—1954, edited by E. S. Pearson and H. O. Hartley and published by Cambridge University Press, London—Reproduced with the kind permission from the Biometrika trustees.

TABLE 1-B

Scale factor, 'c' and equivalent degrees of freedom, 'v' for analysis of double classification, with 'K' blocks and 'n' treatments.

K \ n	2		3		4		5		6		7		8		9	
	V	C	V	C	V	C	V	C	V	C	V	C	V	C	V	C
2	1·0	1·00	2·0	1·35	2·9	1·58	3·8	1·75	4·7	1·89	5·5	2·00	6·3	2·10	7·0	2·18
3	1·9	1·05	3·7	1·48	5·6	1·76	7·4	1·96	9·3	2·12	11·3	2·26	13·4	2·37	15·7	2·46
4	2·7	1·07	5·4	1·54	8·2	1·84	11·0	2·06	13·9	2·23	16·9	2·38	20·1	2·50	23 6	2·60
5	3·6	1·08	7·2	1·57	10·9	1·88	14·6	2·12	18·5	2·30	22·4	2·45	26·6	2·57	31·1	2·68
6	4·5	1·09	8·9	1·59	13 6	1·91	18·2	2·15	23·0	2 34	27·9	2·49	33·0	2·62	38·3	2·73
7	5·4	1·09	10·7	1·61	16·3	1·93	21·8	2·18	27·6	2·37	33·3	2·52	39·3	2·65	45·4	2·76
8	6·3	1·10	12·5	1·62	19 0	1·95	25·4	2·20	32·1	2·39	38·7	2·55	45·6	2 68	52·5	2·79
9	7·1	1·10	14·3	1·63	21·7	1·06	29·0	2·21	36 6	2·41	44·0	2·57	51·8	2·70	59 6	2·81
10	8·1	1·10	16·1	1·63	24·4	1·97	32·6	2·22	41·0	2 42	49·3	2·58	57 9	2·71	66·6	2·83
20	16·7	1·11	33·9	1·66	51·5	2·02	68·8	2.28	86·0	2·48	103·0	2·64	119·0	2·78	134 0	2·90
dn.		1·13		1·69		2·06		2·33		2·53		2 70		2·85		2·97
C. D.	0·87		1·80		2·71		3·62		4·50		5·33		6·10		6.79	

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TABLE II

Percentage points of the Studentized range, $q = (X_n - X_1)/Sv$. Upper 5% points.

V \ n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	18.00	27.0	32.8	37.1	40.4	43.1	45.4	47.4	49.1	50.6	52.0	53.2	54.3	55.4	56.3	57.2	58.0	58.8	59.6
2	6.09	8.3	9.8	10.9	11.7	12.4	13.0	13.5	14.0	14.4	14.7	15.1	15.4	15.7	15.9	16.1	16.4	16.6	16.8
3	4.50	5.91	6.82	7.50	8.04	8.48	8.85	9.18	9.46	9.72	9.95	10.15	10.35	10.52	10.69	10.84	10.98	11.11	11.24
4	3.93	5.04	5.76	6.29	6.71	7.05	7.35	7.60	7.83	8.03	8.21	8.37	8.52	8.66	8.79	8.91	9.03	9.13	9.23
5	3.64	4.60	5.22	5.67	6.03	6.33	6.58	6.80	6.99	7.17	7.32	7.47	7.60	7.72	7.83	7.93	8.03	8.12	8.21
6	3.46	4.34	4.90	5.31	5.63	5.89	6.12	6.32	6.49	6.65	6.79	6.92	7.03	7.14	7.24	7.34	7.43	7.51	7.59
7	3.34	4.16	4.68	5.06	5.36	5.61	5.82	6.00	6.16	6.30	6.43	6.55	6.66	6.76	6.85	6.94	7.02	7.09	7.17
8	3.26	4.04	4.53	4.89	5.17	5.40	5.60	5.77	5.92	6.05	6.18	6.29	6.39	6.48	6.57	6.65	6.73	6.80	6.87
9	3.20	3.95	4.42	4.76	5.02	5.24	5.43	5.60	5.74	5.87	5.98	6.09	6.19	6.28	6.36	6.44	6.51	6.58	6.64
10	3.15	3.88	4.33	4.65	4.91	5.12	5.30	5.46	5.60	5.72	5.83	5.93	6.03	6.11	6.20	6.27	6.34	6.40	6.47
11	3.11	3.82	4.26	4.57	4.82	5.03	5.20	5.35	5.49	5.61	5.71	5.81	5.90	5.99	6.06	6.14	6.20	6.26	6.33
12	3.08	3.77	4.20	4.51	4.75	4.95	5.12	5.27	5.40	5.51	5.62	5.71	5.80	5.88	5.95	6.03	6.09	6.15	6.21
13	3.06	3.73	4.15	4.45	4.69	4.88	5.05	5.19	5.32	5.43	5.53	5.63	5.71	5.79	5.86	5.93	6.00	6.05	6.11
14	3.03	3.70	4.11	4.41	4.64	4.83	4.99	5.13	5.25	5.36	5.46	5.55	5.64	5.72	5.79	5.95	5.92	5.97	6.03
15	3.01	3.67	4.08	4.37	4.60	4.78	4.94	5.08	5.20	5.31	5.40	5.49	5.58	5.65	5.72	5.79	5.85	5.90	5.96
16	3.00	3.65	4.05	4.33	4.56	4.74	4.90	5.03	5.15	5.26	5.35	5.44	5.52	5.59	5.66	5.72	5.79	5.84	5.90
17	2.98	3.63	4.02	4.30	4.52	4.71	4.86	4.99	5.11	5.21	5.31	5.39	5.47	5.55	5.61	5.68	5.74	5.79	5.84
18	2.97	3.61	4.00	4.28	4.49	4.67	4.82	4.96	5.07	5.17	5.27	5.35	5.43	5.50	5.57	5.63	5.69	5.74	5.79
19	2.96	3.59	3.98	4.25	4.47	4.65	4.79	4.92	5.04	5.14	5.23	5.32	5.39	5.46	5.53	5.59	5.65	5.70	5.75
20	2.95	3.58	3.96	4.23	4.45	4.62	4.77	4.90	5.01	5.11	5.20	5.28	5.36	5.43	5.49	5.55	5.61	5.66	5.71
24	2.92	3.53	3.90	4.17	4.37	4.54	4.68	4.81	4.92	5.01	5.10	5.18	5.25	5.32	5.38	5.44	5.50	5.54	5.59
30	2.89	3.49	3.84	4.10	4.30	4.46	4.60	4.72	4.83	4.92	5.00	5.08	5.15	5.21	5.27	5.33	5.38	5.43	5.48
40	2.86	3.44	3.79	4.04	4.23	4.39	4.52	4.63	4.74	4.82	4.91	4.98	5.05	5.11	5.16	5.22	5.27	5.31	5.36
60	2.83	3.40	3.74	3.98	4.16	4.31	4.44	4.55	4.65	4.73	4.81	4.88	4.94	5.00	5.06	5.11	5.16	5.20	5.24
120	2.80	3.36	3.69	3.92	4.10	4.24	4.36	4.48	4.56	4.64	4.72	4.78	4.84	4.90	4.95	5.00	5.05	5.09	5.13
	2.77	3.31	3.63	3.86	4.03	4.17	4.29	4.39	4.47	4.55	4.62	4.68	4.74	4.80	4.85	4.89	4.93	4.97	5.01

'n' is the size of sample from which the range is obtained and 'v' is number of degrees of freedom of 'Sv.'

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